3D GR Hydrodynamic Simulations of Binary Neutron Star Coalescence and Stellar Collapse with Multipatch Grids

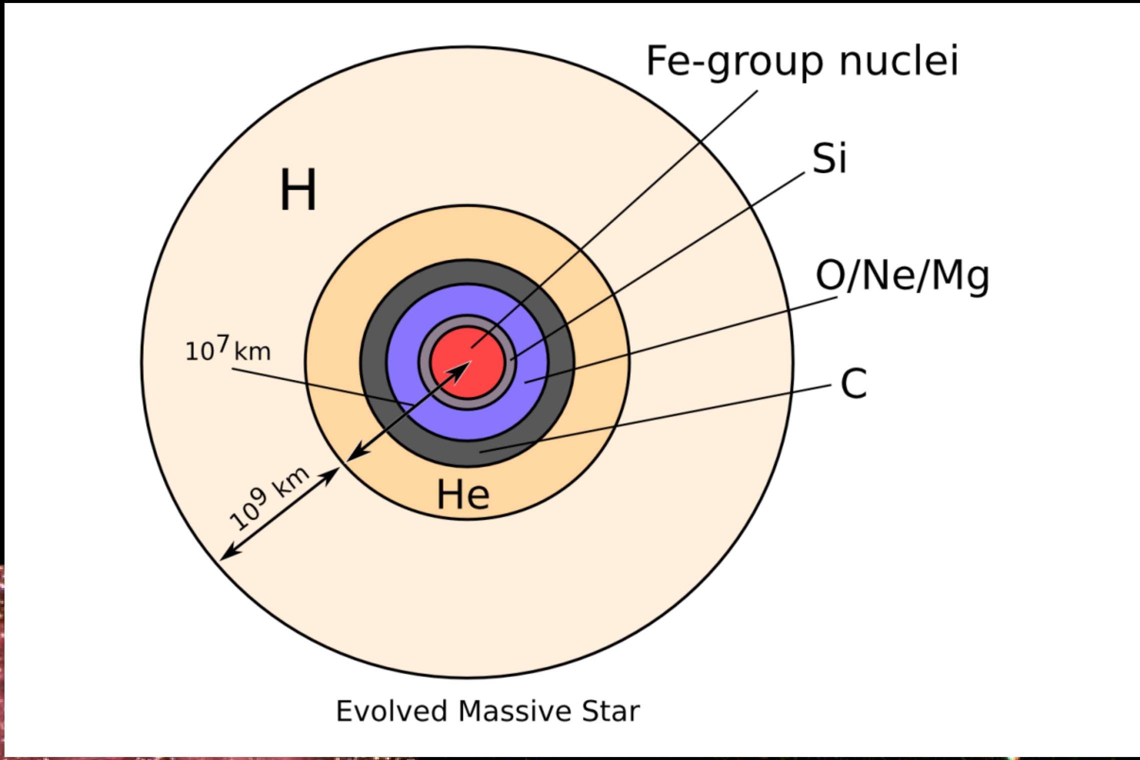
Christian Reisswig
Caltech

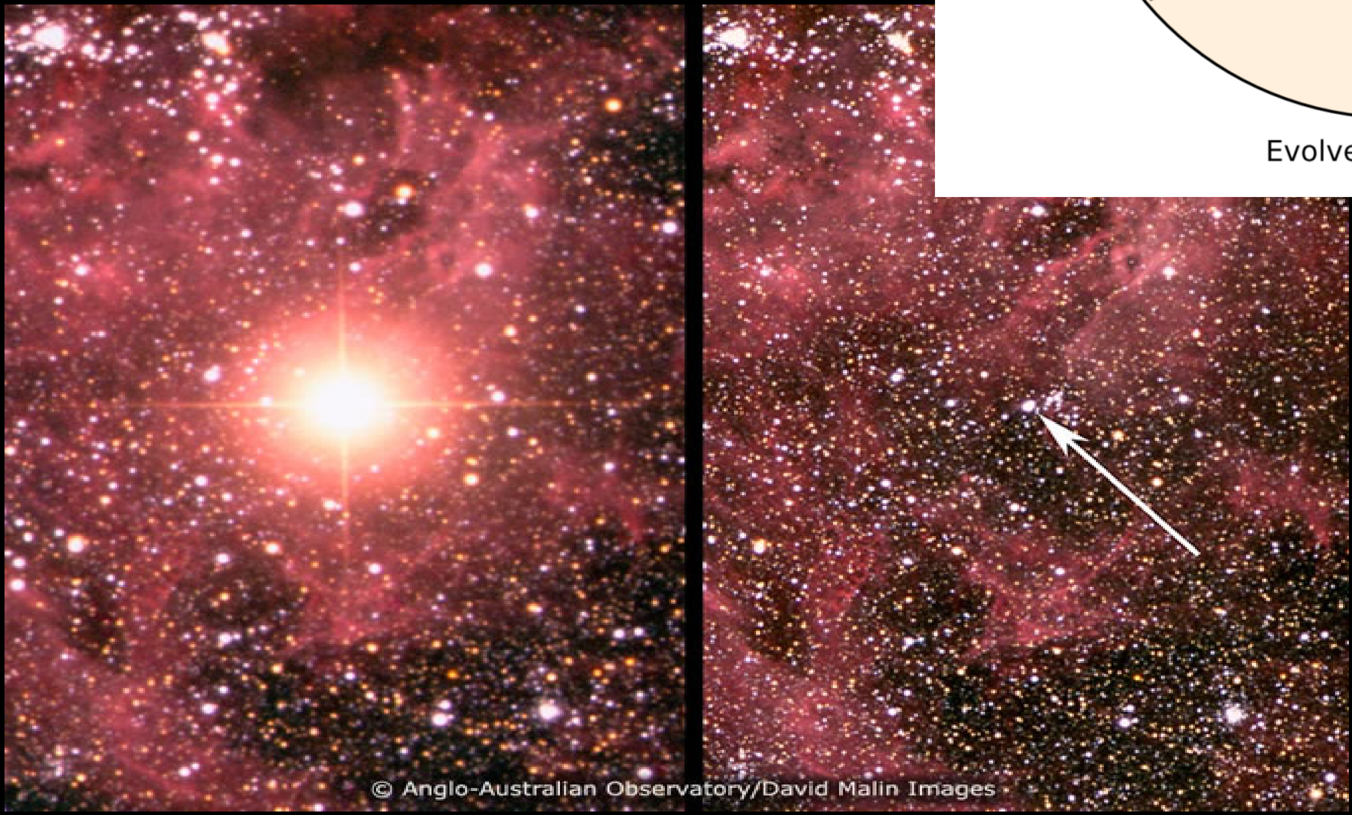
R. Haas, C. D. Ott, E. Abdikamalov, P. Moesta, D. Pollney, E. Schnetter

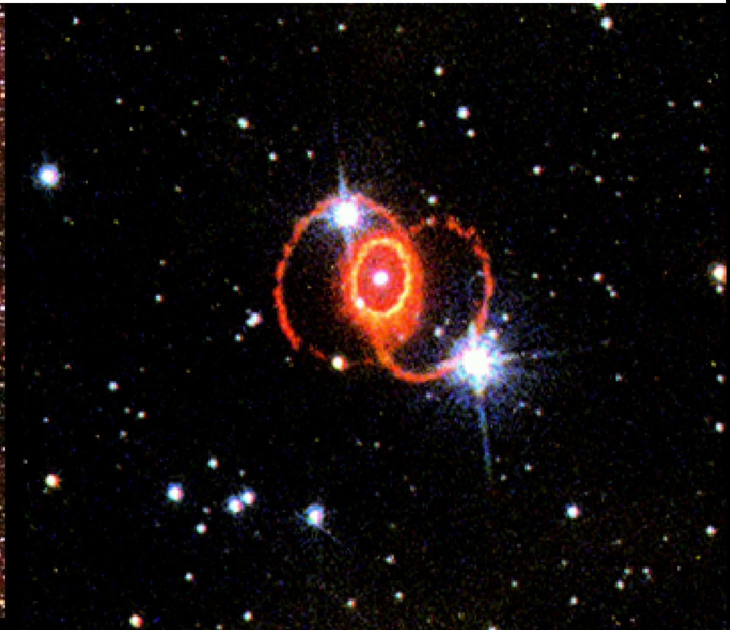
Motivation: Core-Collapse Supernovae

Stellar Collapse!

- Core-collapse supernovae (Type II, Ib/c)
- Neutron stars
- Stellar mass black holes
- Long gamma-ray bursts







(Ideal) Computational Modeling

Magneto-Hydrodynamics	→ Dynamics of stellar fluid
General Relativity	→ Gravity
Nuclear and Neutrino Physics	→ Nuclear EOS, nuclear (neutrino) interactions
Boltzmann Transport Theory	→ Neutrino transport

- Multi-D: convection, turbulence, SASI, rotation (ideally 3D)
- Modeling on massively parallel computers (>1000-10000 cores)

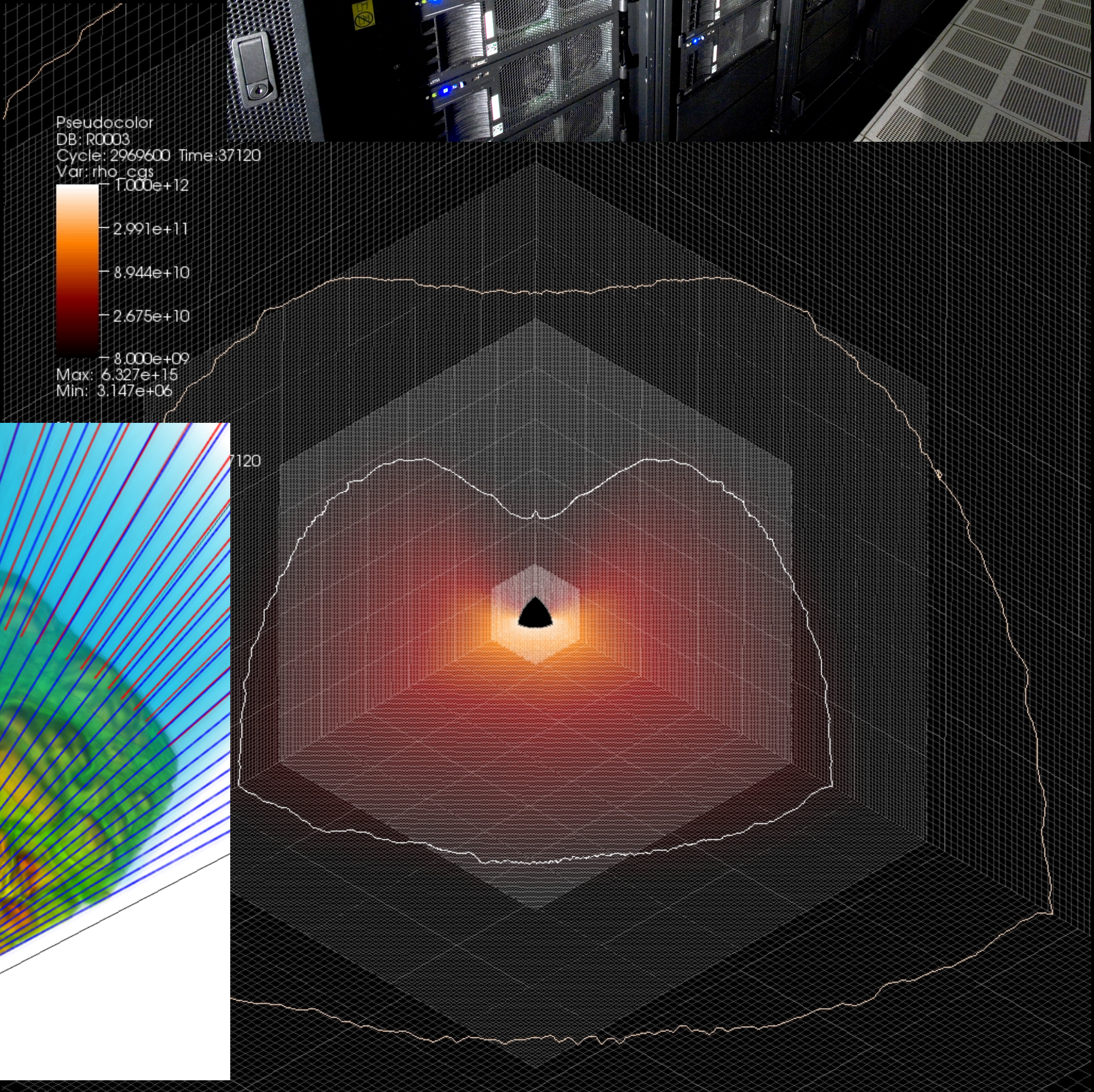
Adaptive mesh refinement, task-based parallelism, 3D Monte-Carlo radiation transport, Discontinuous Galerkin Methods...

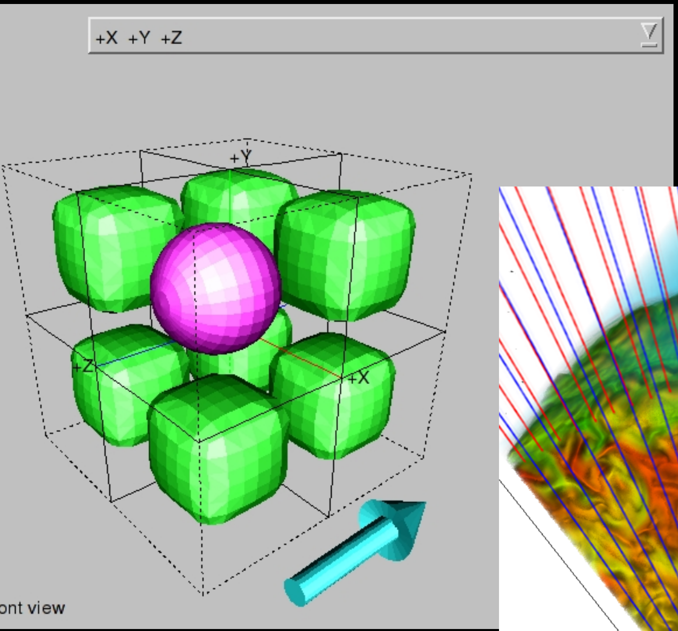
EXTREMELY CHALLENGING!

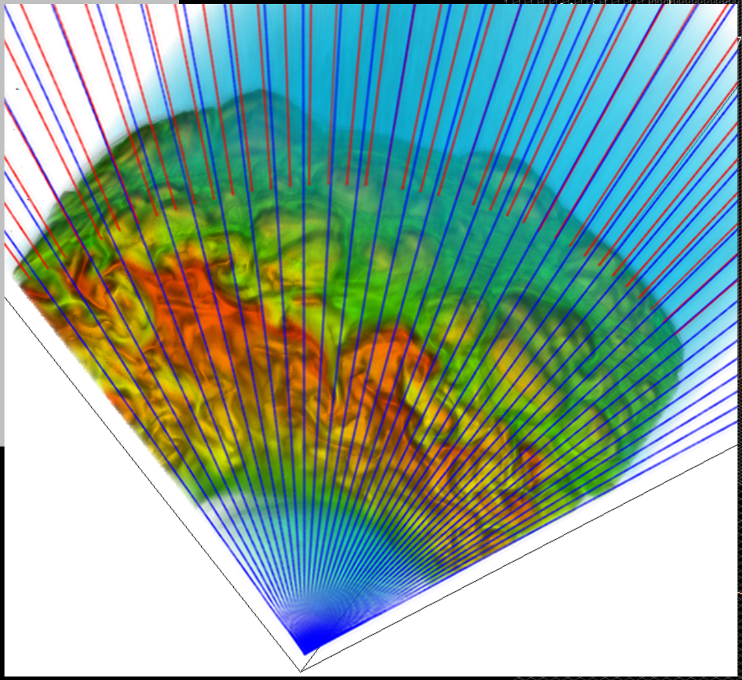
→ Most studies so far: 1D and 2D!

Computational Modeling

- General Relativistic Hydrodynamics (GRHydro)
- Finite Volume Scheme with adaptive mesh refinement (Carpet)
- Realistic equation of state
- Neutrino Transport (leakage scheme)
- 3D (octant symmetry)







Motivation: Binary Neutron Stars

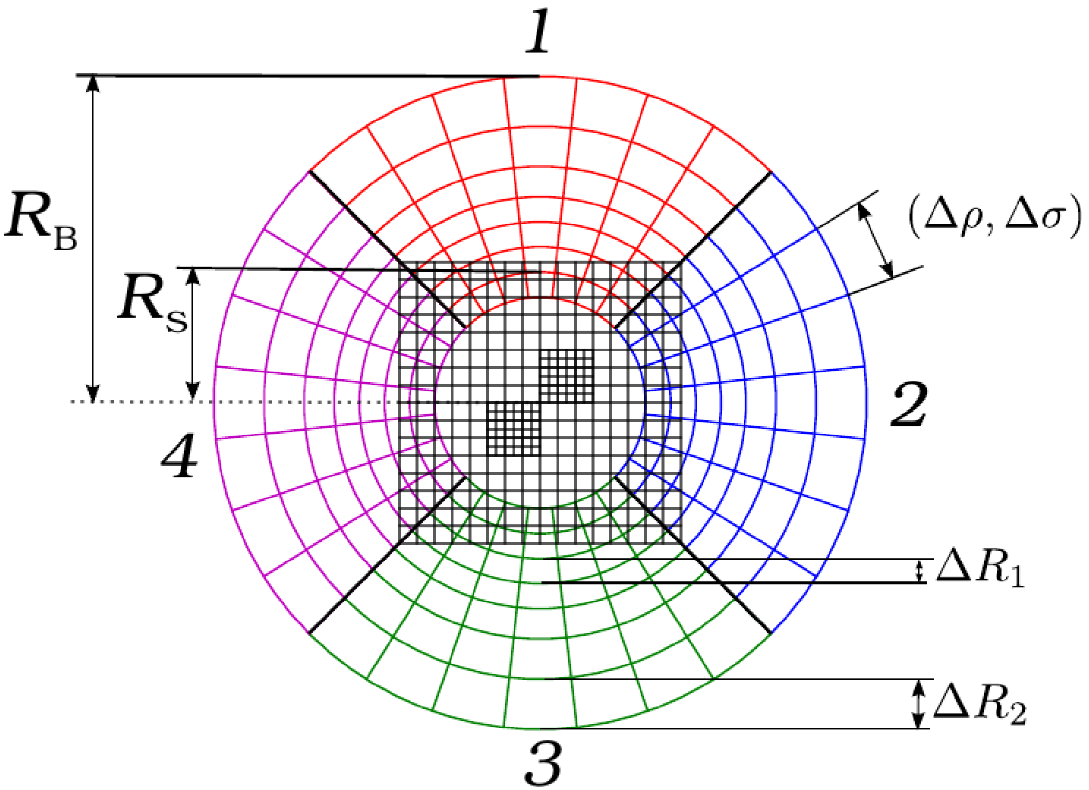
- Excellent source for GWs!
- GW signal will yield valuable info about EOS
- Could power sGRBs

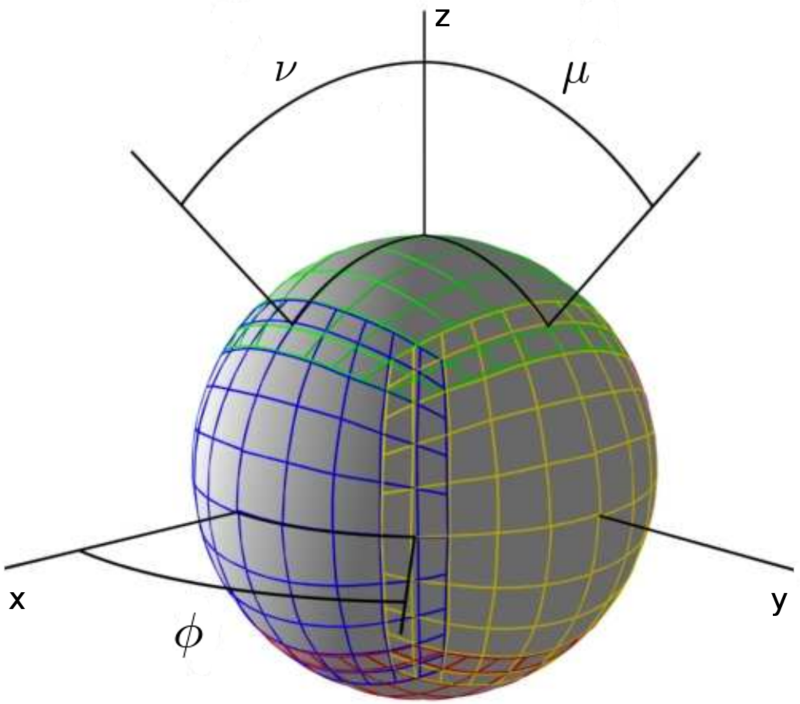
- Large high-resolution wave-extraction zone would allow to resolve higher modes

- Larger computational domains would allow to track ejected (unbound) material → r-processes, EM counterparts?

Core-Collapse Supernovae

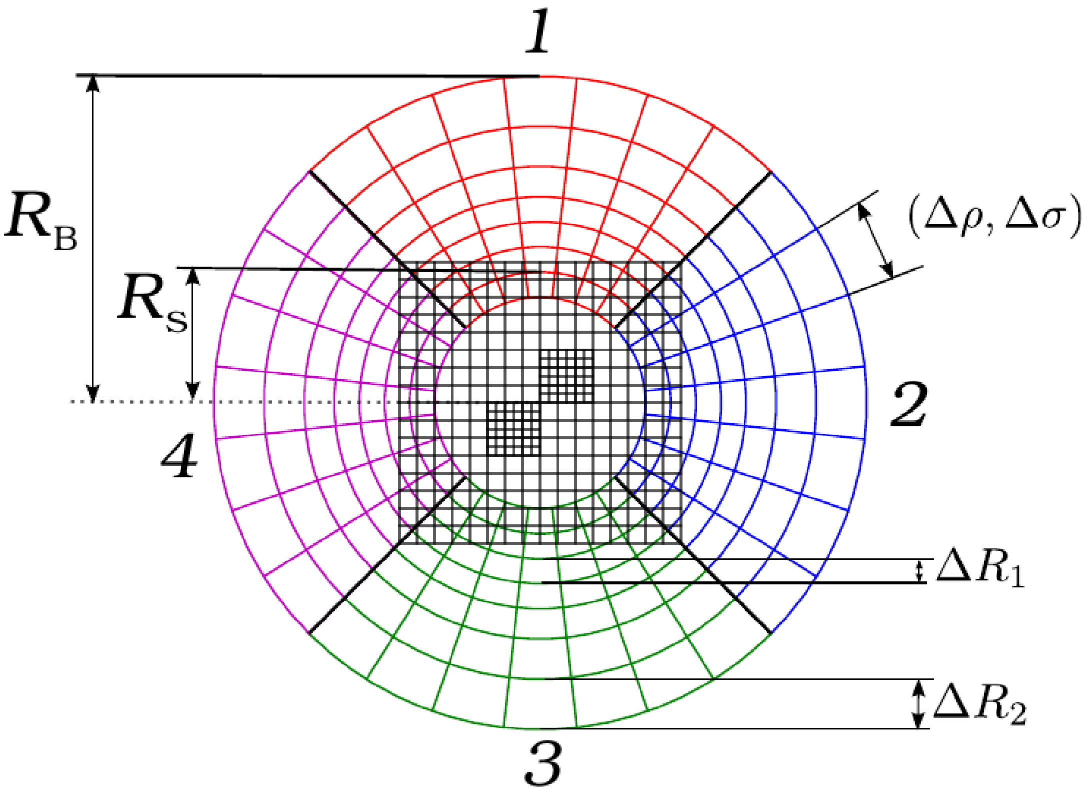
- How can we improve our modeling? How can we go to full unconstraint 3D?
- Possible solution: Multiblocks

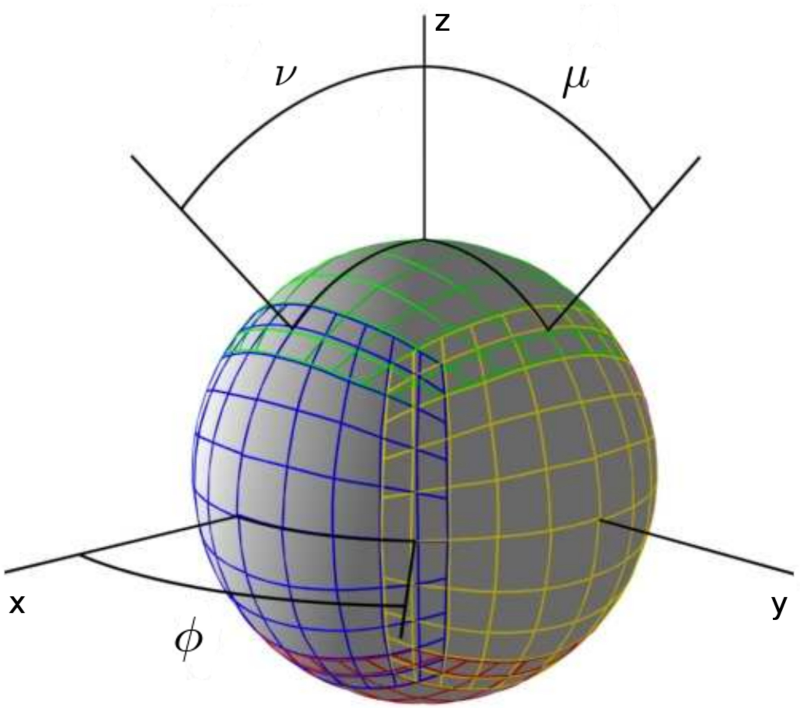




Binary Neutron Stars

- How can we improve GW extraction / enlarge the domains?
- Possible solution: Multiblocks





Multiblocks

- A set of curvilinear grid patches covers the domain

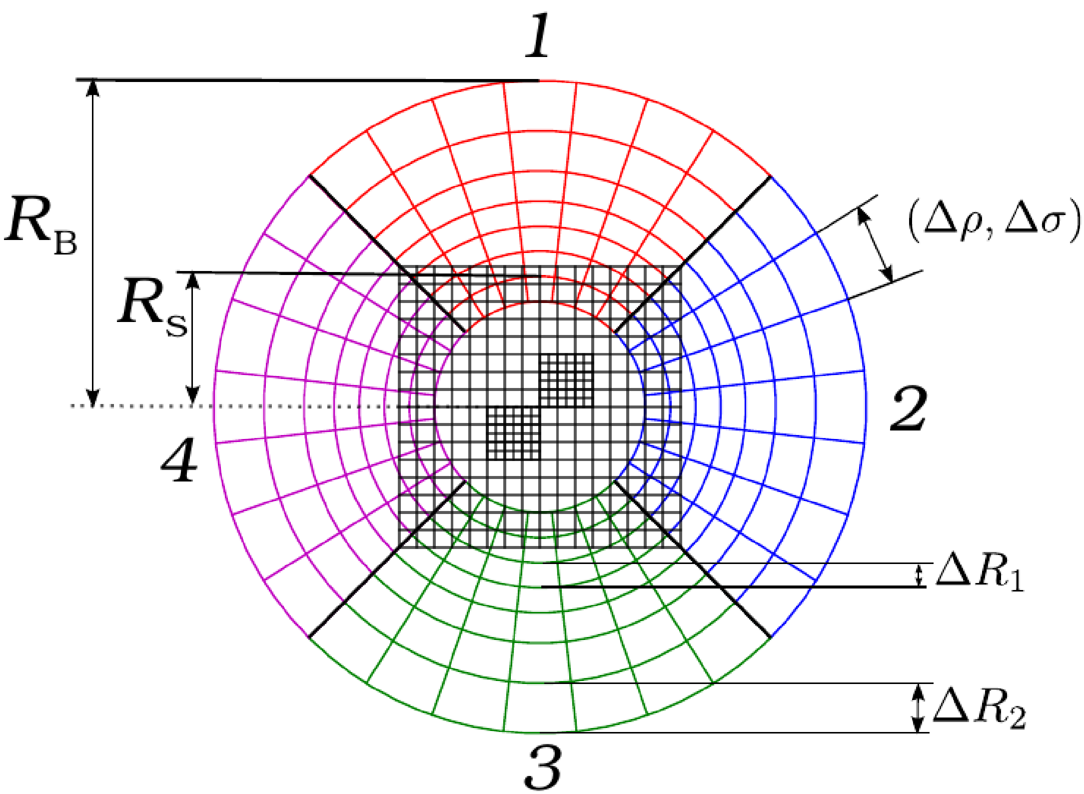
  ⟶ Grids can be adapted to problem symmetry

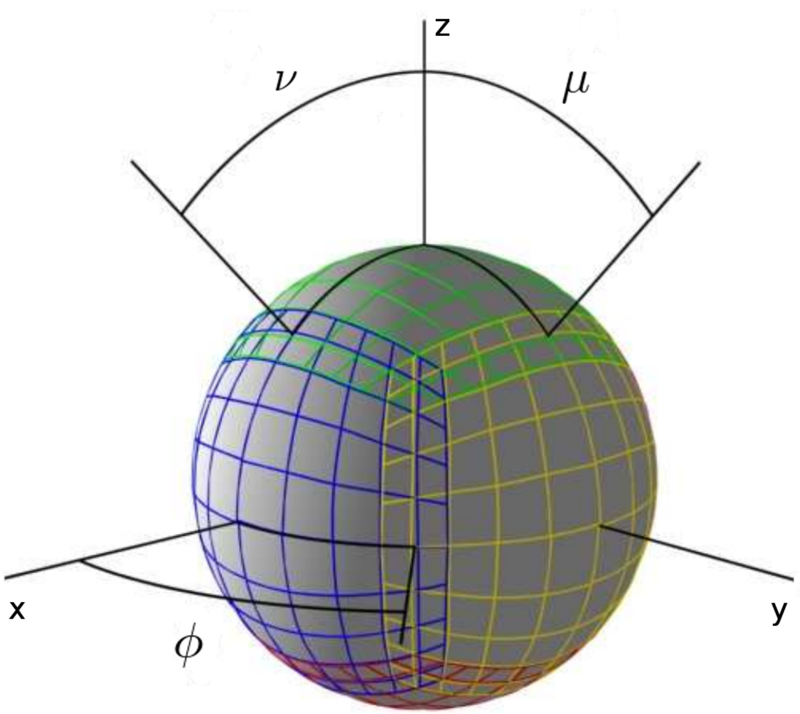
- Useful patch system: Central Cartesian patch with AMR

  Spherical grids for exterior region

Inflated-cube grid

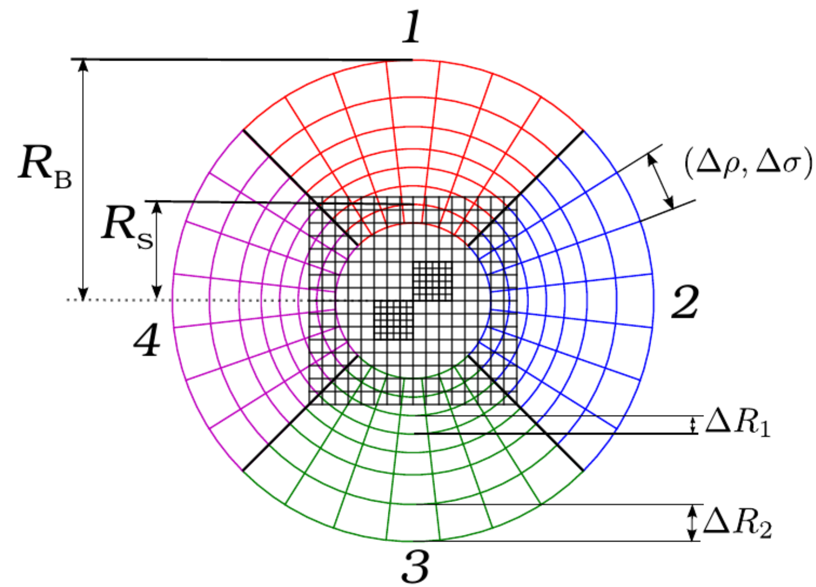
Radial stretching





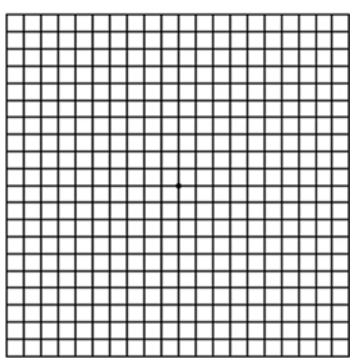
Multiblocks

- Each grid patch is locally Cartesian



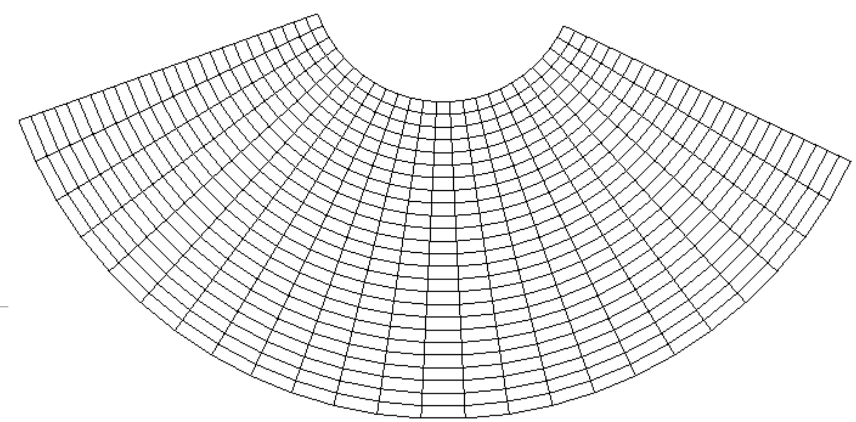
Generic Strategy

- Solve fluid evolution in local coordinates, curvature evolution in global coordinates
- Coupling in global tensor basis



$$\frac{\partial x^\mu}{\partial u^\nu} \longrightarrow$$

$$\longleftarrow \frac{\partial u^\mu}{\partial x^\nu}$$



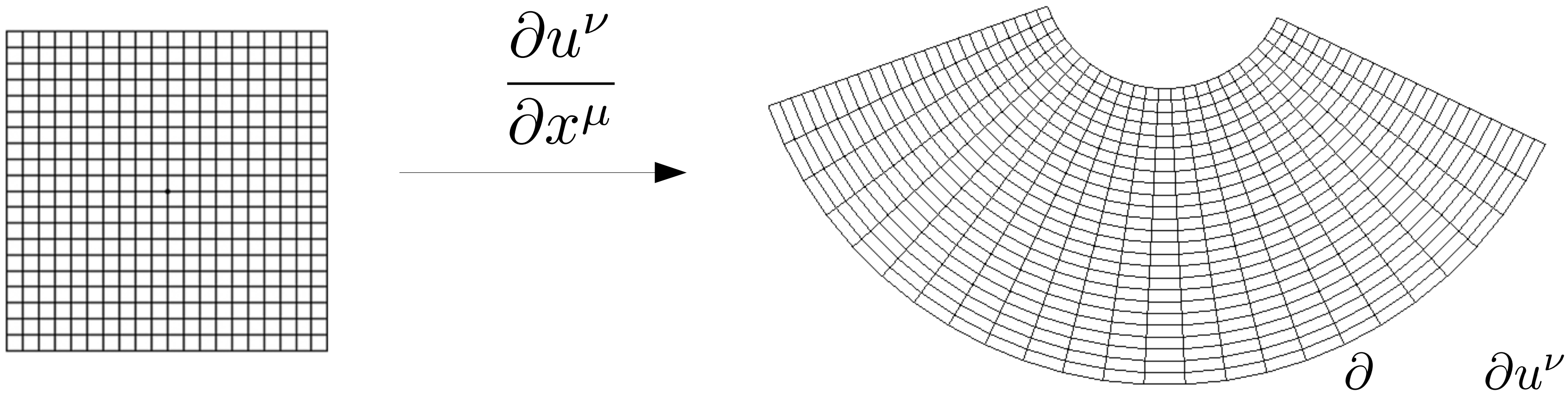
$$g_{\mu\nu},\ T_{\mu\nu} \qquad\qquad \tilde{g}_{\mu\nu},\ \tilde{T}_{\mu\nu}$$

Need Jacobian transformations to transform between local and global frame

Multiblocks: Spacetime Solver

- Finite difference derivatives approximated in local basis:

$$\frac{\partial}{\partial u^\nu}$$



$$\frac{\partial u^\nu}{\partial x^\mu}$$

$$\frac{\partial}{\partial x^\mu} = \frac{\partial u^\nu}{\partial x^\mu}\frac{\partial}{\partial u^\nu}$$

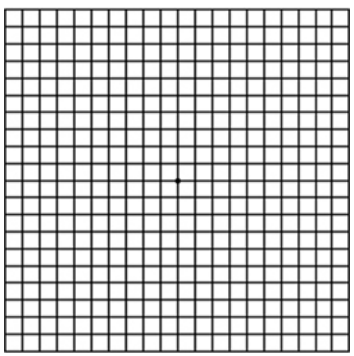
- Evolution equations are evaluated in global basis
- Can keep original Cartesian code; only need to replace derivative operators!

Pollney et al, Phys.Rev.D 83, 2011

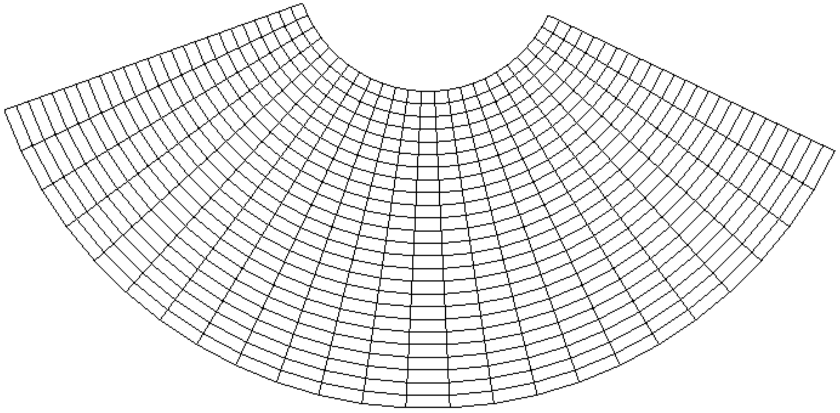
Multiblocks: Hydro Solver

- Hydrodynamic equations are solved via HRSC finite volume method (GRHydro)
- GRHydro is based on uniform grids

solve hydro eqns. in local basis (where grids are uniform!)

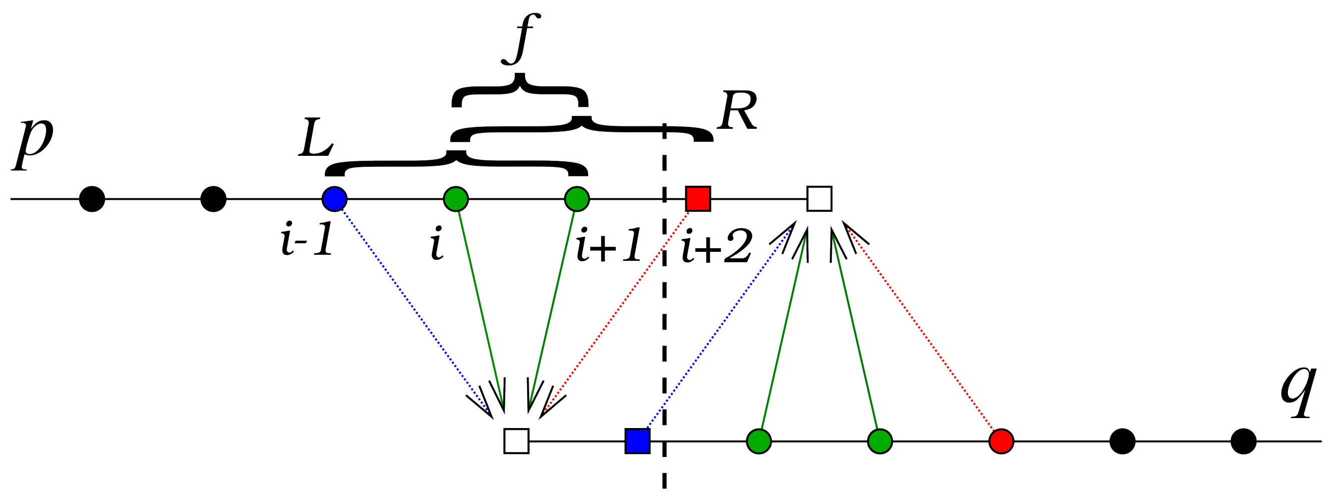


$$\frac{\partial u^{\nu}}{\partial x^{\mu}}$$



Multiblocks: Interpatch interpolation

- Information at patch boundaries exchanged via inter-patch interpolation (Lagrange / ENO2)



- Grid patches need to overlap to ensure interpolation from nominal points

- Tensorial quantities may need to be transformed

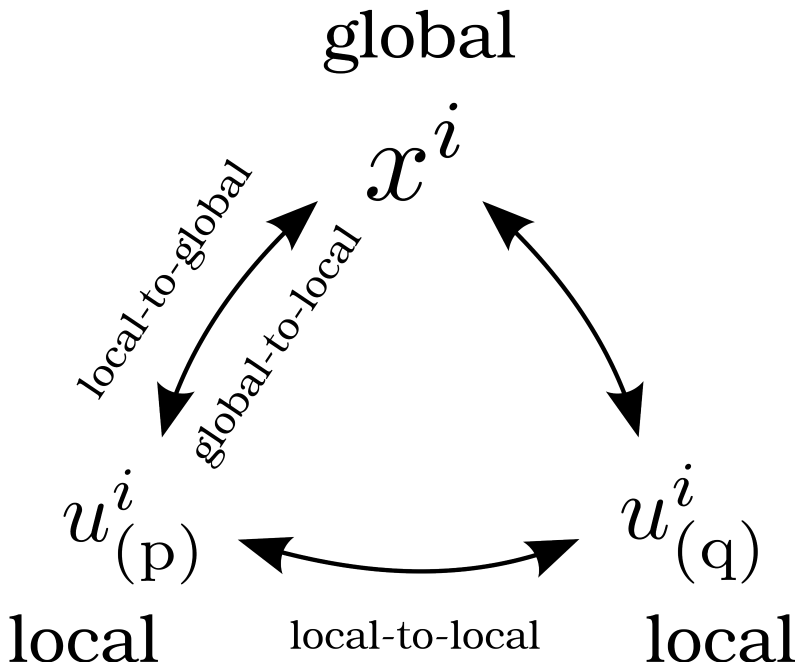
Implementation: Hydro

- Thorn EinsteinEvolve/GRHydro:

  Conserved variables in local basis

  Primitive variables in local basis

  ADM metric and shift in local basis



- EinsteinBase/HydroBase: primitive variables in global basis

- 1) ADM metric and shift: global-to-local transform
- 2) Hydro interpatch interpolation: local-to-local transform
- 3) Primitives and stress-energy tensor: local-to-global transform

Implementation: Multipatch

- Thorn Llama/Coordinates
- Sets up patch system and coordinate descriptions
- Carpet is responsible for management of maps (memory allocation of Gfs)
- Stores Jacobians, inverse Jacobians and derivatives of Jacobians

$$\frac{\partial u^{\nu}}{\partial x^{\mu}} \qquad \frac{\partial u^{\mu}}{\partial x^{\nu}}$$

- Custom patch systems can be “easily” added

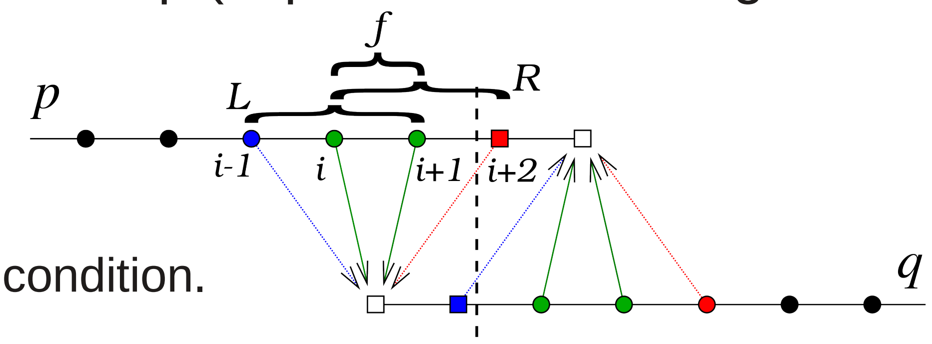
Implementation: Interpolation

- Interpolation via thorn Llama/Interpolate2:

  Sets up Carpet/CarpetInterp2 interpolation data structures

  Applies coordinates transformation after interpolation

- CarpetInterp2: Stores interpolation coefficients for each interpolation point.
- Tree-based search to speed up coordinate lookup (important when using many processors)



Interpolation is registered as a symmetry boundary condition.

Variables are interpatch synchronized via SelectBC, ApplyBC mechanism.

Other modeling improvements

- Cell-centered AMR
- Flux conservation at AMR boundaries
- Multirate Runge-Kutta scheme (RK2-RK4)
- Enhanced piecewise parabolic reconstruction

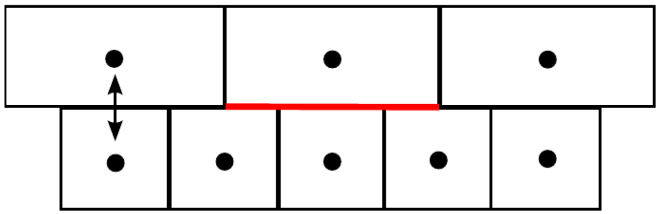
→ Improved numerical efficiency / accuracy!

- Optimized synchronization: Don't sync everything!

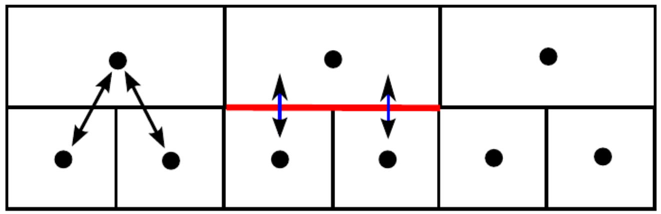
→ Improved scaling!

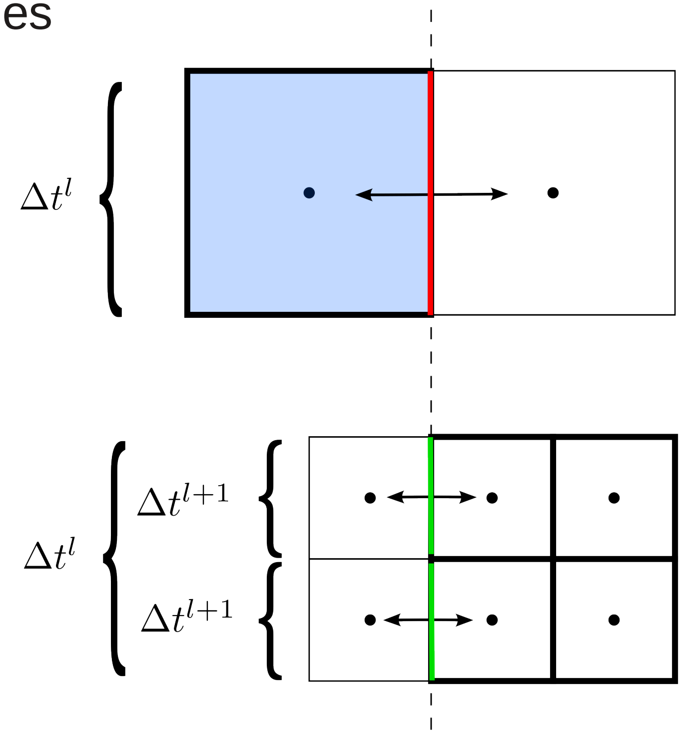
Cell-centered AMR / Refluxing

Refluxing ensures conservation at AMR boundaries



Requires cell-centered AMR!





1) Capture fluxes on coarse and fine grid AMR boundary

2) Integrate both until coarse and fine grid are aligned in time again

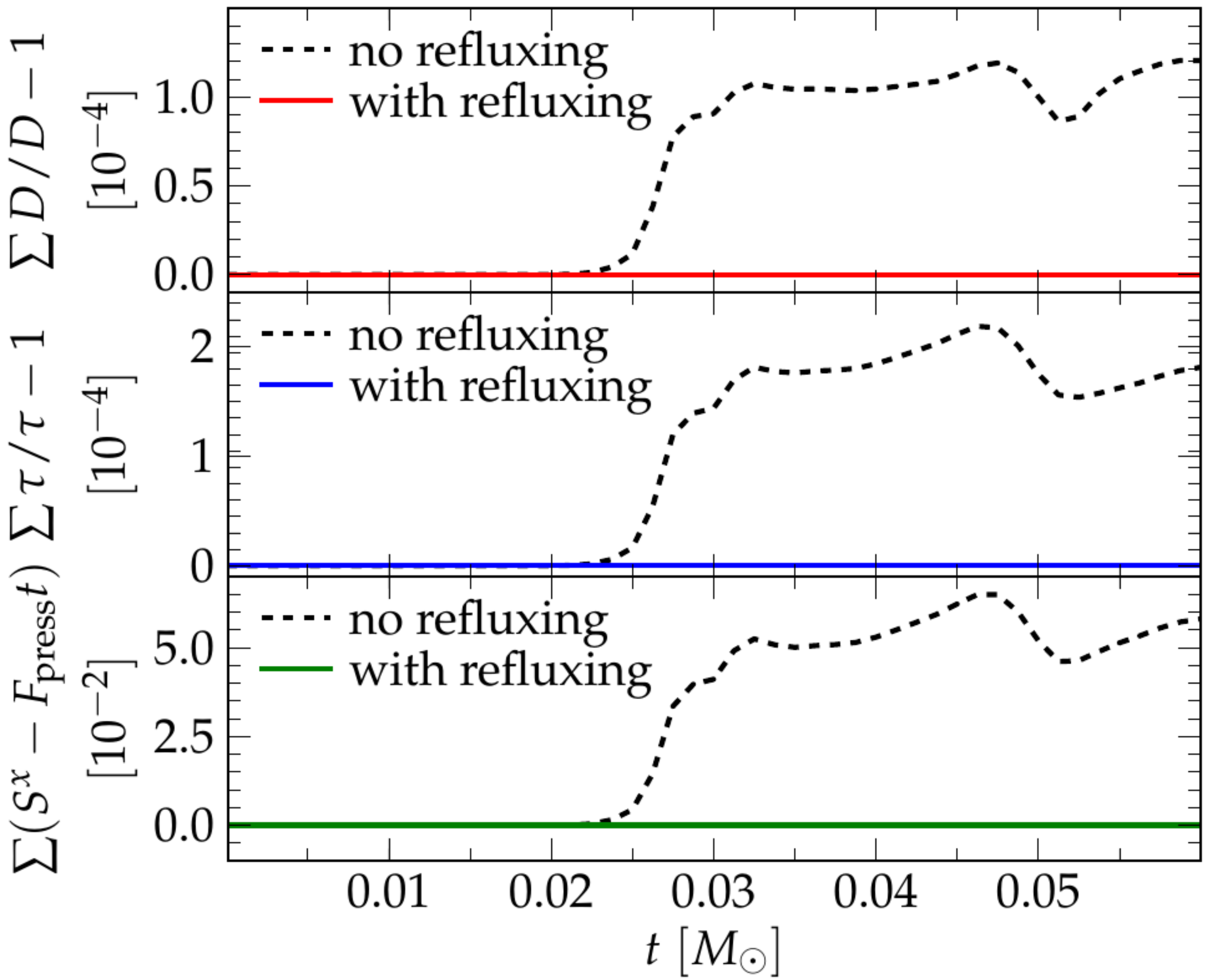
3) Restrict integrated coarse grid flux onto fine grid boundary

4) Difference between integrated coarse grid flux and fine grid flux is correction

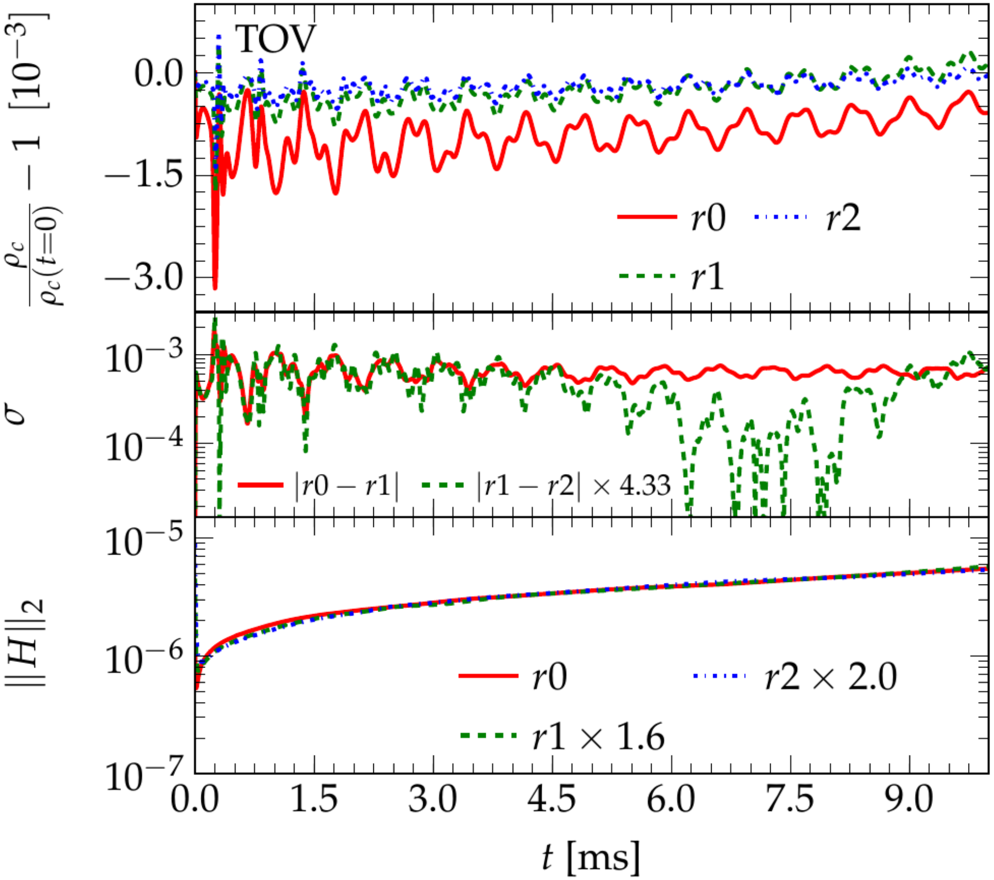
Cell-centered AMR / Refluxing

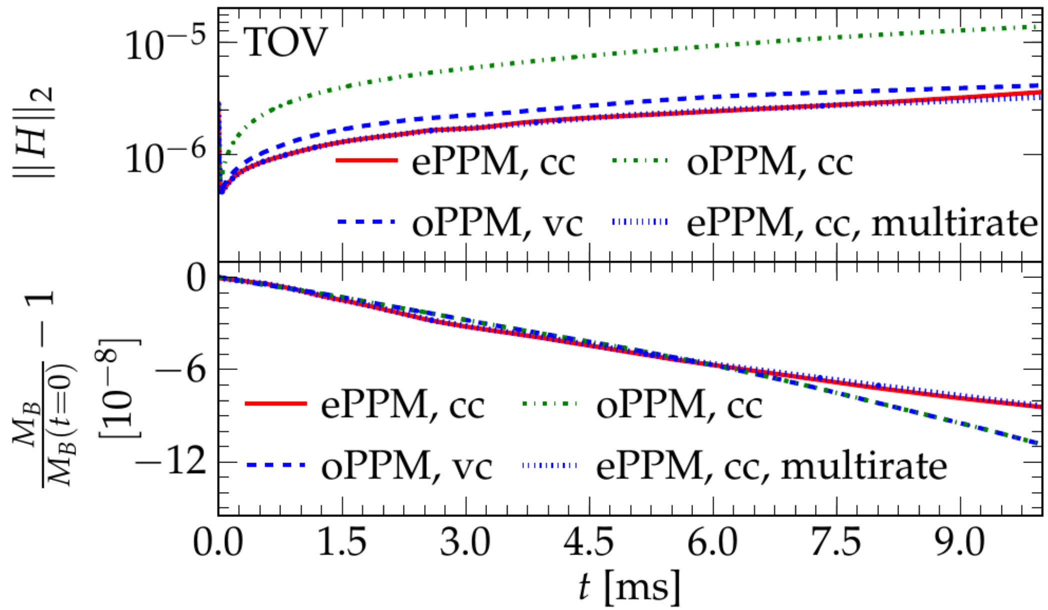
Refluxing ensures conservation at AMR boundaries

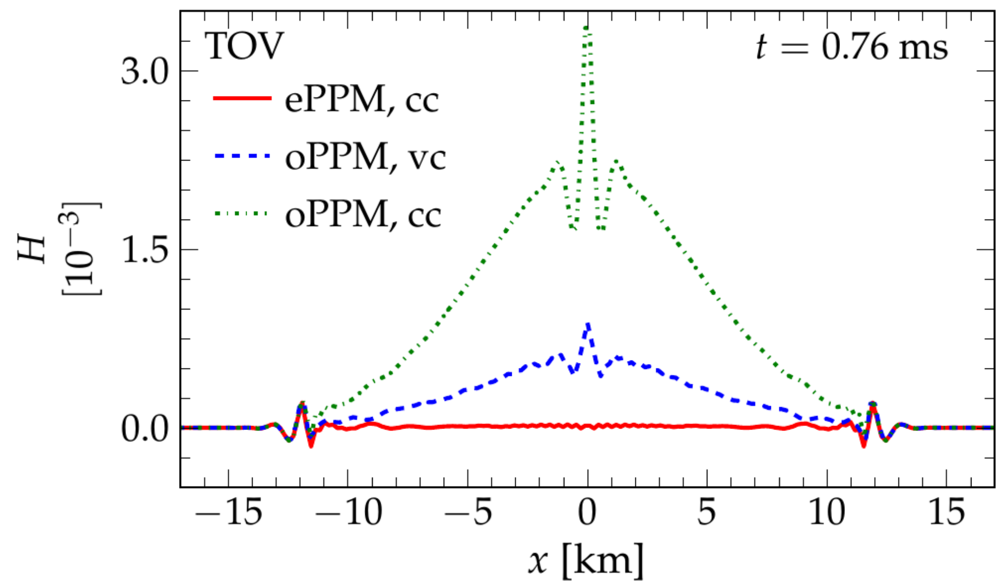
Shock front moving from fine to coarse grid



Tests: TOV star



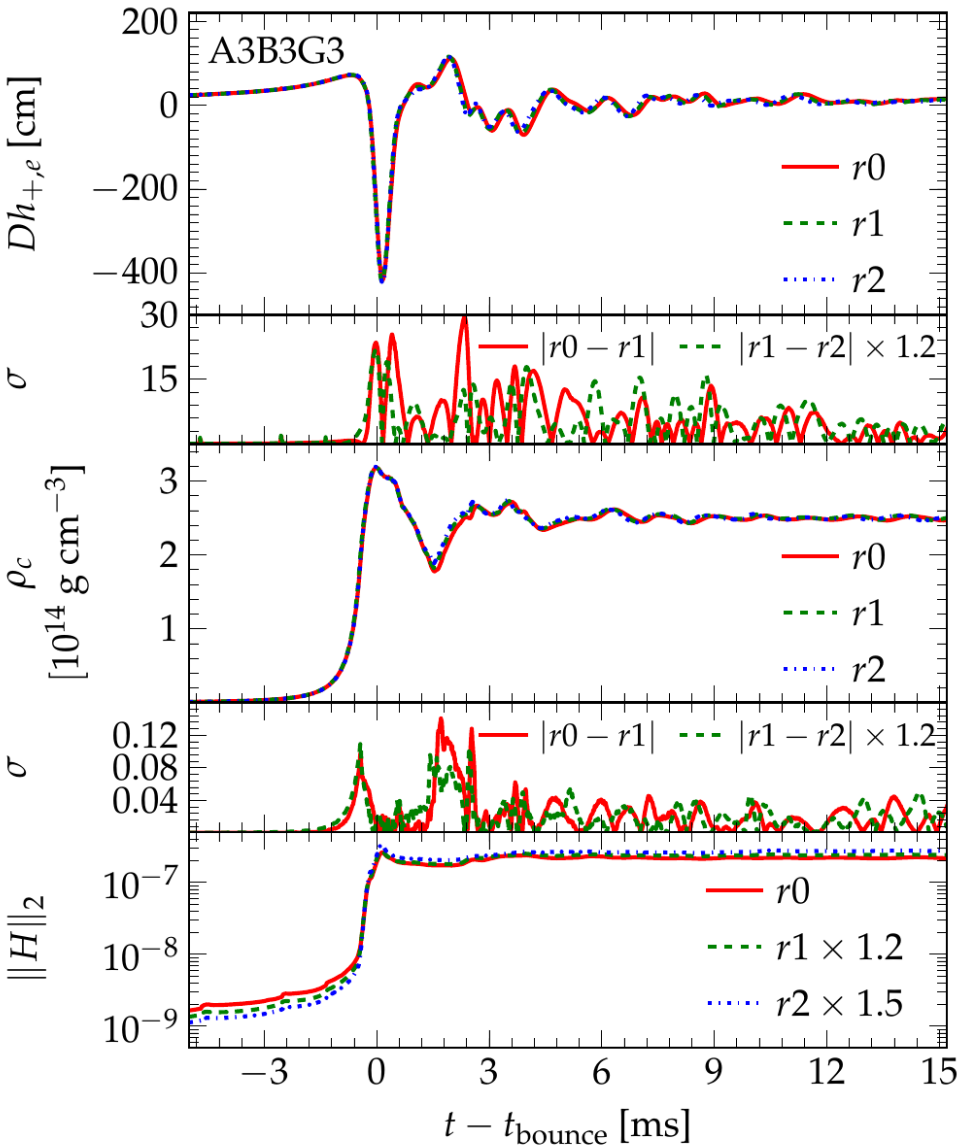


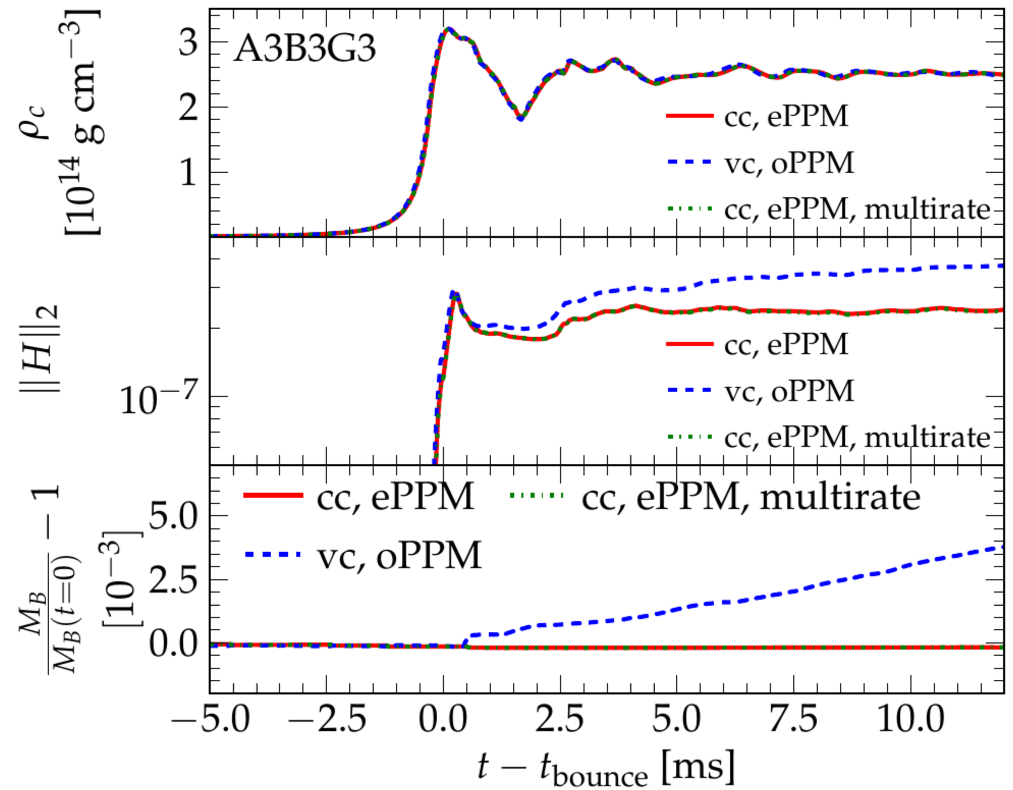


K=100, Gamma=2 Polytrope

Ideal gas EOS

Tests: Stellar Core Collapse



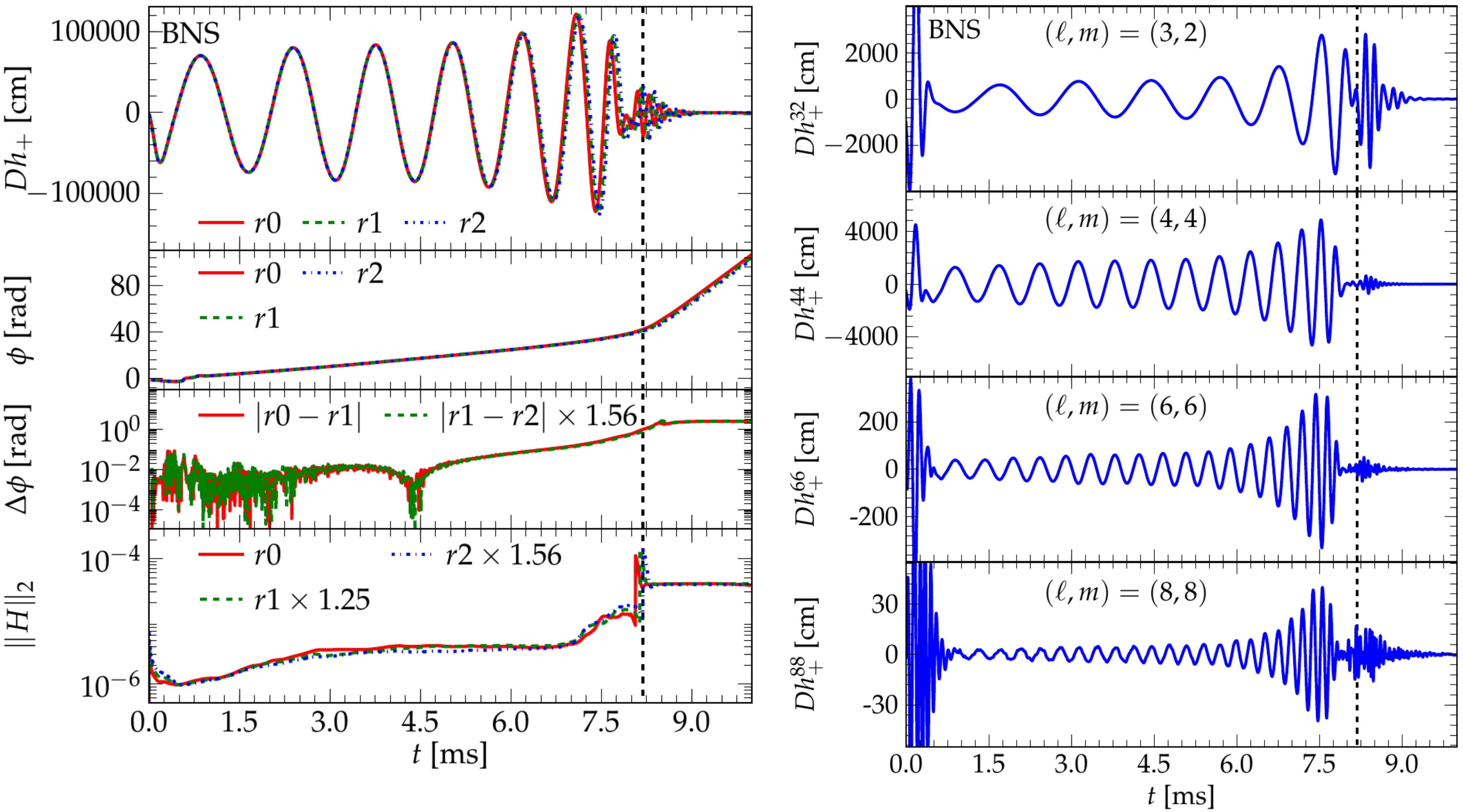


Hybrid EOS

Rapid Differential Rotation
→ Strong Core Bounce Signal

- Interpatch boundaries threading star
- Inner core on Cartesian grid

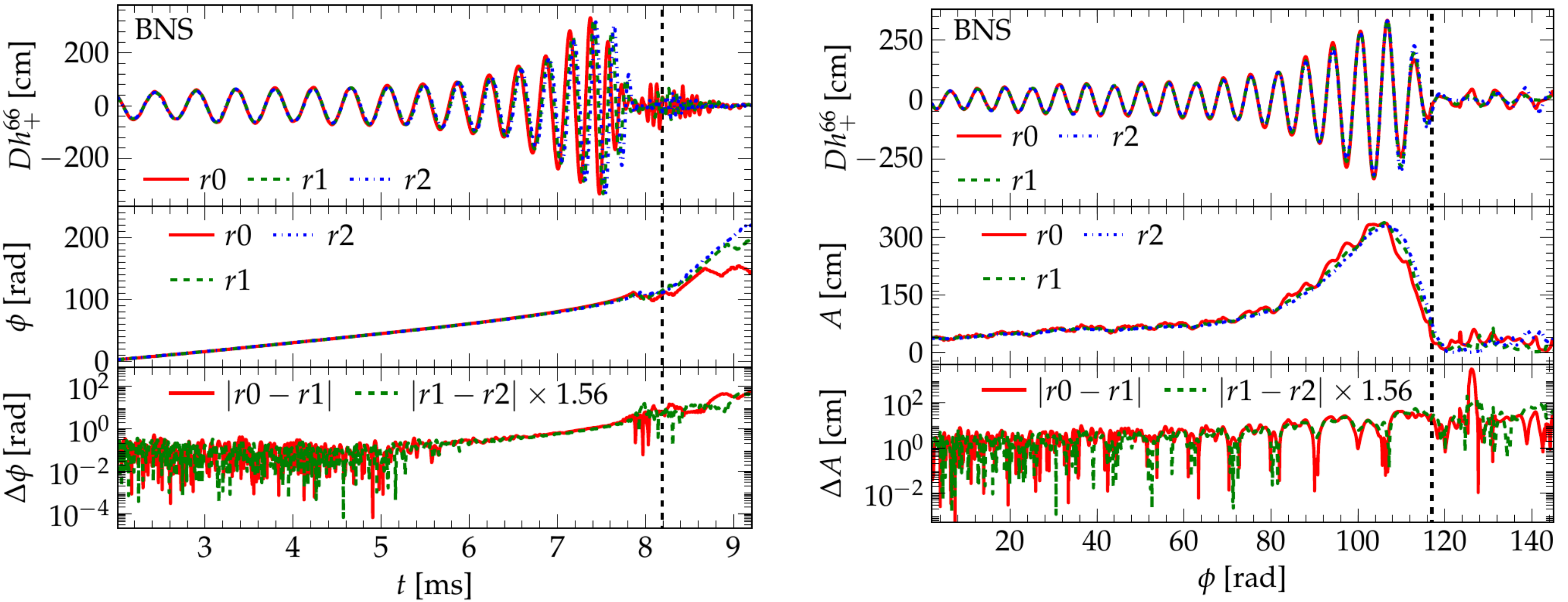
Tests: Binary Neutron Stars



Wave-extraction via Cauchy-characteristic extraction at Scri+

Tests: Binary Neutron Stars

Convergence (l,m)=(6,6)



Summary

- We have implemented a new multiblock scheme for more efficient 3D general relativistic hydro simulations
- Higher accuracy in GW extraction can be achieved with multiblocks
- Cell-centered AMR + Refluxing greatly help to ensure the conservative properties of the scheme
- Codes are being made / are publicly available as part of the ET

Reisswig et al 2012, arXiv:1212.1191