Improved EM Gauge Condition for GRMHD Simulations with AMR

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Introduction

- Illinois Numerical Relativity group recently developed a full GR MHD code capable of evolving the Einstein-Maxwell-MHD system of coupled equations in 3+1 dimensions in AMR [Phys. Rev. D **82**, 084031 (2010)]

- Evolve the vector potential **A** (ensure that $\nabla\cdot\mathbf{B}=0$)

- Perform interpolation on **A** between AMR refinement levels

- Performed many tests to validate the code (shock-tube tests, magnetized Bondi accretion, collapse of magnetized rotating relativity star)

- First application: magnetized black hole-neutron star (BHNS) simulations

- Subtle issue: EM gauge

MHD Equations

GR	Newtonian

Equations for gravitational field

$$G_{\mu\upsilon} = 8\pi T_{\mu\upsilon}$$

$$\nabla^2\Phi = -4\pi\rho$$

Maxwell's equations in MHD limit

$$\partial_j\left(\sqrt{\gamma}B^j\right) = 0$$

$$\partial_t(\sqrt{\gamma}B^i) + \partial_j\left[\sqrt{\gamma}\left(\mathrm{v}^j B^i - \mathrm{v}^i B^j\right)\right] = 0$$

$$\nabla\cdot\mathbf{B} = \mathbf{0}$$

$$\partial_t\mathbf{B} = \nabla\times(\mathbf{v}\times\mathbf{B})$$

Fluid equations

$$\partial_t\rho_* + \partial_j\left(\rho_*\mathrm{v}^j\right) = 0$$

$$\partial_t\tilde{S}_i + \partial_j\left(\alpha\sqrt{\gamma}T^j{}_i\right) = \frac{1}{2}\alpha\sqrt{\gamma}T^{\alpha\beta}\partial_i g_{\alpha\beta}$$

$$\partial_t\tilde{\tau} + \partial_j\left(-n_\mu\alpha\sqrt{\gamma}T^{\mu i} - \rho_*\mathrm{v}^j\right) = s$$

$$\partial_t\rho + \nabla\cdot\left(\rho\mathbf{v}\right) = 0$$

$$\rho\left(\partial_t\mathbf{v} + \mathbf{v}\cdot\nabla\mathbf{v}\right) = -\nabla\left(P + \frac{B^2}{8\pi}\right) + \frac{\mathbf{B}\cdot\nabla\mathbf{B}}{4\pi} - \rho\nabla\Phi$$

$$\rho\left(\partial_t\varepsilon + \mathbf{v}\cdot\nabla\varepsilon\right) + P\nabla\cdot\mathbf{v} = 0$$

Magnetic Constraint and Induction Equation

$$\nabla \cdot \mathbf{B} = 0 \quad , \quad \partial_t \mathbf{B} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

$$\Rightarrow \quad A^\mu = (\Phi, \mathbf{A}) \qquad \mathbf{B} = \nabla \times \mathbf{A} \qquad \partial_t \mathbf{A} = \mathbf{v} \times \mathbf{B} - \nabla\Phi$$

"algebraic" gauge: $\Phi = \text{const.}$
$\Rightarrow \quad \partial_t \mathbf{A} = \mathbf{v} \times \mathbf{B}$

Eigenmode analysis

EM evolution equations decouple from MHD and Einstein equations in short-wavelength, small-amplitude (harmonic) perturbations about a background solution.

$$\partial_t \mathbf{A} = \mathbf{v} \times (\nabla \times \mathbf{A}) \quad , \quad \nabla \to \mathbf{k} \ \text{(wavevector with } |\mathbf{k}|=1\text{)}$$

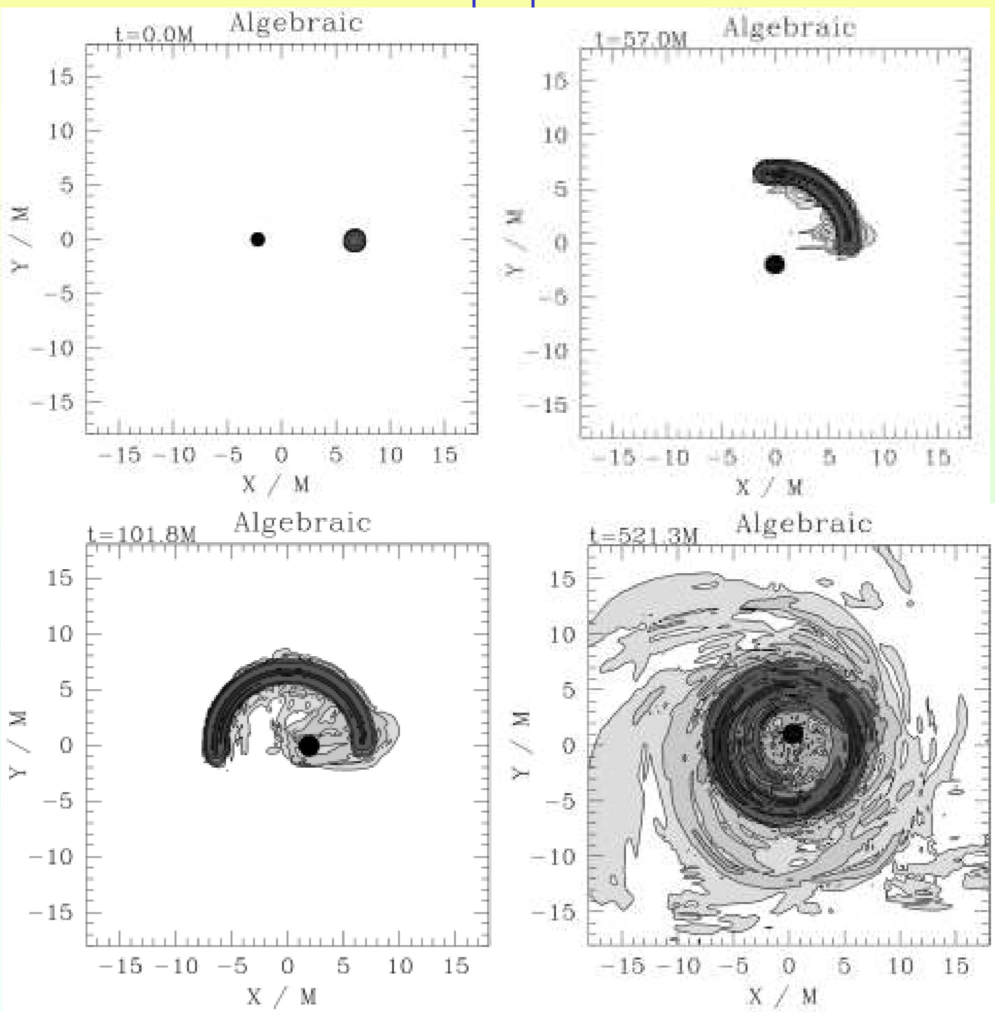
$$\Rightarrow \quad \partial_t \mathbf{A} = \mathbf{M}\mathbf{A} \quad , \quad M_{ij} = k_i \mathrm{v}_j - (\mathbf{v} \cdot \mathbf{k})\delta_{ij}$$

Eigenvalues of $\mathbf{M}$: $\lambda_1 = 0,\ \ \lambda_2 = \lambda_3 = \mathbf{v} \cdot \mathbf{k}$

Zero-speed mode!

Magnetized BHNS Simulations in “Algebraic” Gauge with AMR

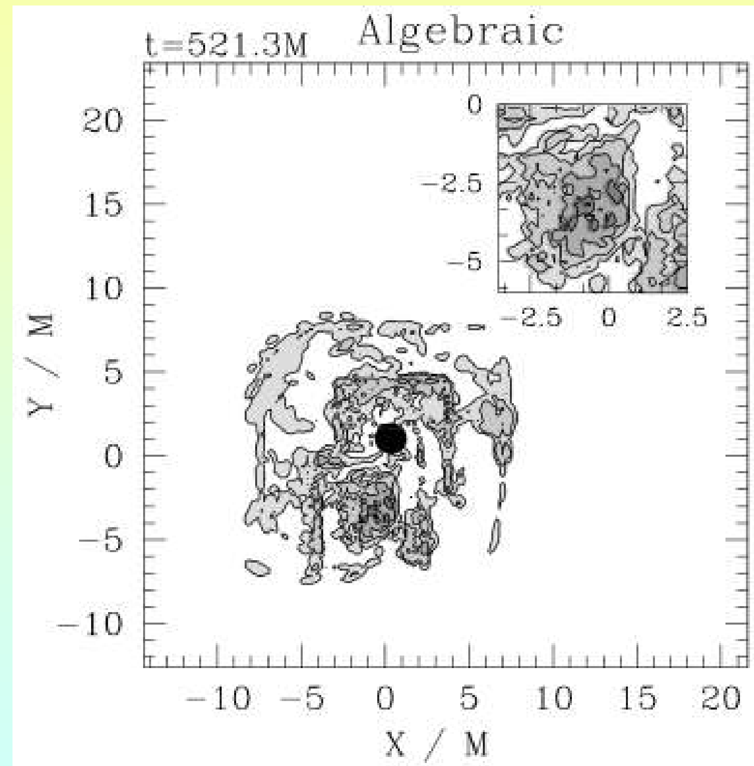
$|\mathbf{A}|^2$



Trail causes numerical problem?

Unigrid: no AMR: yes!

magnetic energy density



Spurious B-fields develop!

Affect regions both inside and outside NS!

Lorenz Gauge

Lorenz gauge condition: $\nabla_\mu A^\mu = 0 \quad \Rightarrow \quad \partial_t \Phi + \nabla \cdot \mathbf{A} = 0$

Induction equation: $\partial_t \mathbf{A} = \mathbf{v} \times (\nabla \times \mathbf{A}) - \nabla \Phi$

Eigenmode analysis

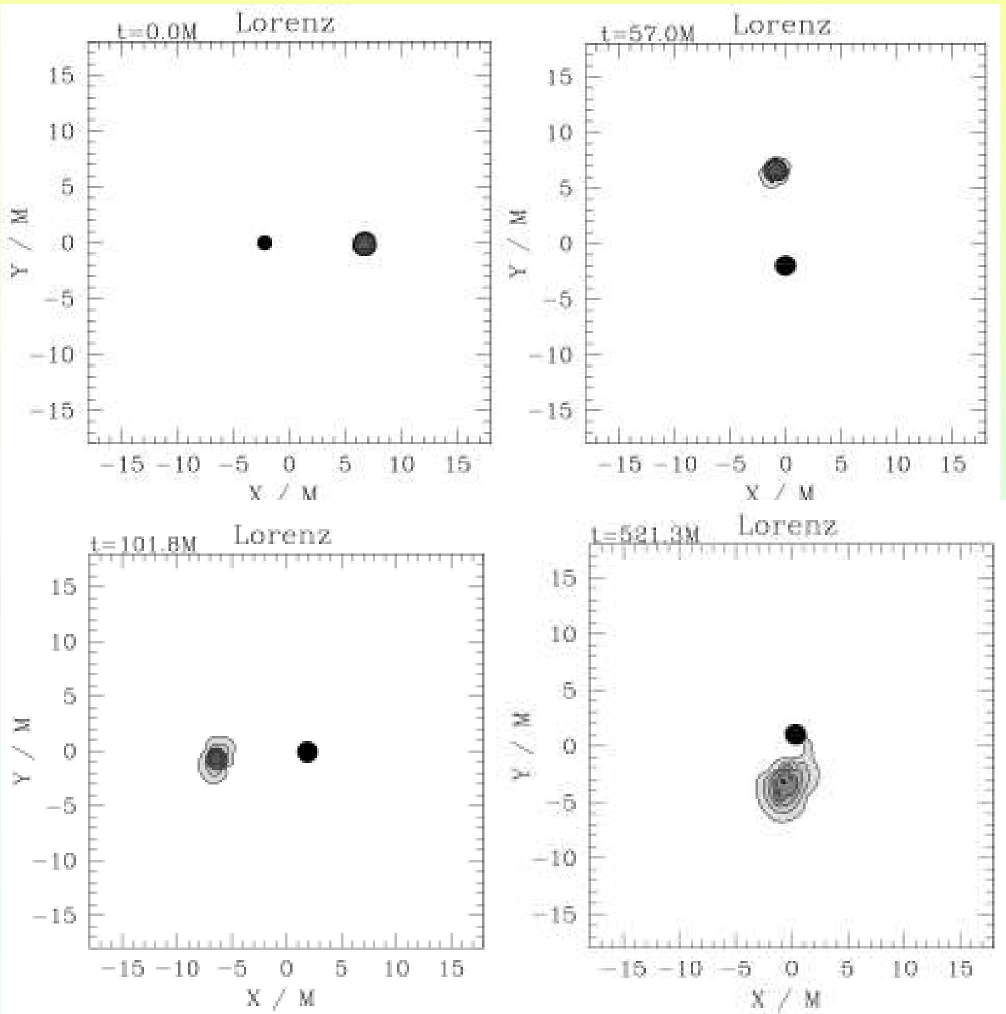
$$\nabla \to \mathbf{k} \quad \Rightarrow \quad \partial_t A_\mu = M_{\mu\upsilon} A^\upsilon$$

Eigenvalues of $\mathbf{M}$: $\lambda_{1,2} = \pm 1,\ \lambda_3 = \lambda_4 = \mathbf{v} \cdot \mathbf{k}$

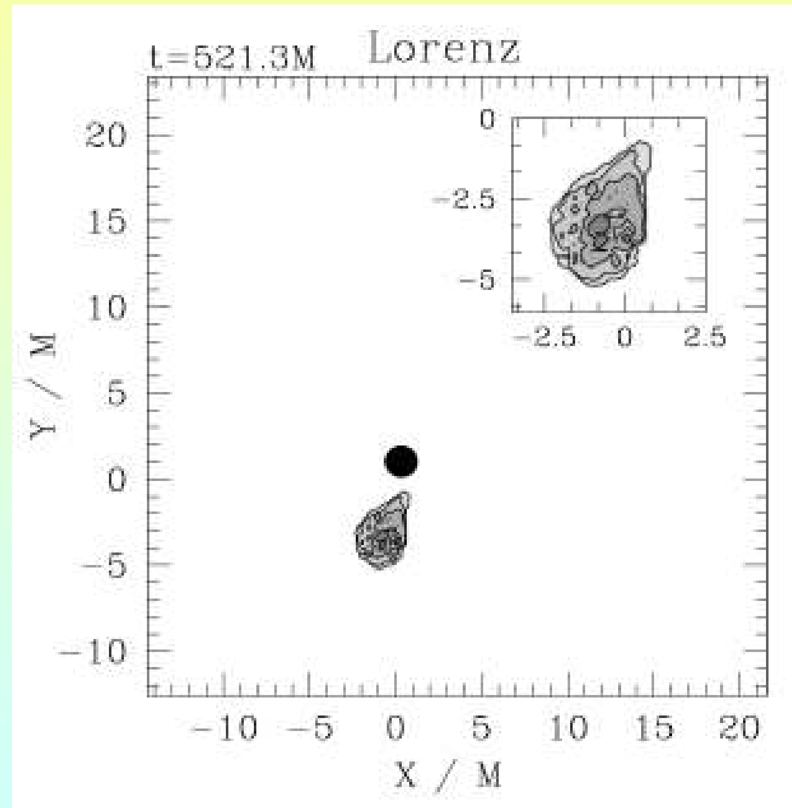
No zero-speed mode!

Magnetized BHNS Simulations in Lorenz Gauge

$|\mathbf{A}|^2$



magnetic energy density



Spurious B-field greatly reduced!

Summary

- Developed a new GRMHD code compatible with AMR
- Evolve vector potential **A** (ensure that $\nabla \cdot \mathbf{B}=0$)
- Zero-speed mode in "algebraic" gauge gives spurious B-fields in magnetized BHNS simulations with AMR
- Spurious B-fields are significantly reduced in Lorenz gauge
- Performed magnetized BHNS simulations using Lorenz gauge (Etienne et al, in preparation)